Revisiting the Case for the Endowment Model

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Following strong relative US equity performance during the period 2009 - 2014, many investors have questioned the efficacy of the endowment model relative to traditional asset allocation strategies with higher weightings to US assets (for example, 60% US equities, 40% US-centric fixed income). In this paper, we assume that an investor constructing a mean-variance efficient portfolio as of December 31, 2009 possesses a degree of clairvoyance regarding the joint distribution of asset class returns (mean, variance, and covariance) over the subsequent five years. Despite the apparent adversity of such a scenario for the endowment model, we find that this process is more likely to lead to endowment-like portfolios than 60/40 portfolios, particularly when risk (volatility) is taken into account.

Note that this analysis explores the viability of the endowment model relative to traditional "60/40" portfolios under a set of reasonable but simple assumptions. It is not our intention to propose a specific asset allocation that we believe is optimal, either for past or future periods. Such an undertaking would require a more granular approach to formulating capital market expectations and the integration of client-specific factors, among other things.

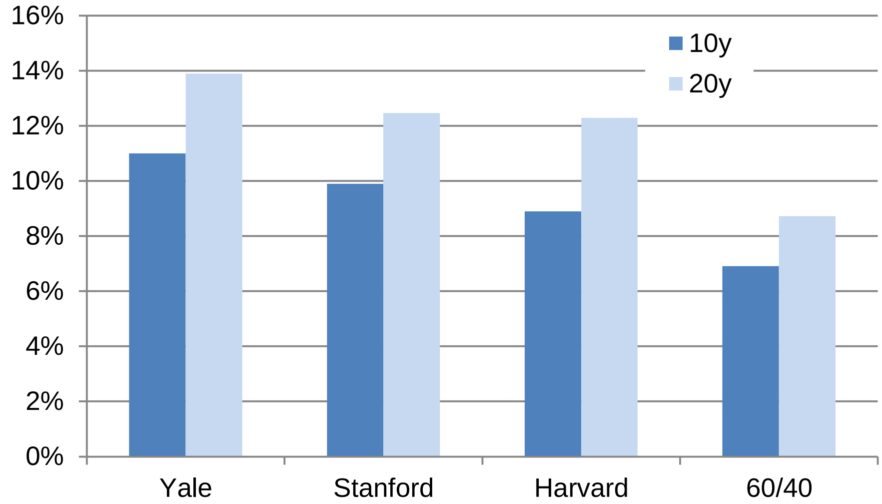
The endowment model of investing was popularized by David Swensen, manager of Yale's endowment since 1985. Under the endowment model, the virtually infinite horizon and soft spending commitments of endowments enable allocations to potentially illiquid and volatile asset classes such as alternative investments, foreign equities, and real assets. The primary benefit of the endowment model is improved risk-adjusted returns, due to the following:

1. **Increased portfolio diversification**: For example, the "risky" portion of the endowment model portfolio is comprised of a variety of asset classes that aren't perfectly correlated, as opposed to being 100% US equities. There is potential for certain risky asset classes to do well when others perform poorly, muting the magnitude of fluctuations in the overall portfolio.

2. **Access to market inefficiencies and unique sources of return**: Investors are able to capture alpha generated by skilled experts (hedge funds, private equity, and private real estate) and additional sources of investment return (i.e. the liquidity premium) that aren't available through the simple 60/40 structure.

Over long periods of time, leading university endowments have outperformed the 60/40, both on a risk-adjusted return and risk-indifferent return basis. Exhibit 1 shows historical returns for Yale, Harvard and Stanford as of June 2014, with the 60/40 portfolio on the right.

Exhibit 1[1,2,3,4]
Portfolio Returns for 10 and 20 Year Periods
(through June 2014)



However, in recent years, the endowment model has underperformed. It has been pointed out that, since the crisis of 2008, an investor might have done better with the traditional 60/40 portfolio. As shown in Exhibit 2, this is largely due to the underperformance of hedge funds, foreign equities, and commodities, whereas US equities have thrived. Private equity and private real estate also performed strongly during this period.

[1] Yale Office of Public Affairs & Communications. "Investment return of 20.2% brings Yale endowment value to $23.9 billion." Yale News, September 2014, http://news.yale.edu/2014/09/24/investment-return-202-brings-yale-endowment-value-239-billion

[2] Stanford Management Company. "Stanford Management Company 2014 Annual Report." Stanford University, June 2014, http://www.smc.stanford.edu/?q=communication
Stanford Management Company. "Stanford Management Company 2004 Annual Report." Stanford University, June 2004, http://www.smc.stanford.edu/?q=communication

[3] Harvard Management Company. "Harvard University endowment delivers 15.4% return for fiscal year 2014." Harvard Gazette, September 2014, http://news.harvard.edu/gazette/story/2014/09/harvard-university-endowment-delivers-15-4-return-for-fiscal-year-2014/

[4] 60/40 is comprised of 60% S&P 500 Index, 40% Barclays Aggregate Bond Index. Endowment returns are presented net of fees, 60/40 is presented gross of fees.*

Exhibit 2

Risk Table Oct 2009 - Sep 2014 Sorted by annualized RoR (total returns)	Annualized RoR	Standard Deviation	Sharpe Ratio	Maximum Drawdown
Cambridge Associates LLC U.S. Private Equity Index	16.63%	5.45%	2.54	-4.32%
FTSE NAREIT Equity REITs Index	16.16%	14.24%	1.00	-15.07%
S&P 500 Index	15.70%	14.43%	0.96	-13.87%
Russell 2000 Index	14.29%	18.77%	0.71	-23.12%
NCREIF - National Index	10.99%	2.71%	3.17	-2.11%
MSCI EAFE Index	7.04%	16.69%	0.37	-18.95%
HFRI Fund Weighted Composite Index	5.05%	5.94%	0.53	-7.63%
MSCI Emerging Markets Index	4.76%	18.03%	0.24	-23.27%
Barclays Aggregate Bond Index	4.13%	3.04%	0.70	-2.45%
Bloomberg Commodity Index	-1.37%	15.55%	-0.14	-29.84%

Where does this leave us? Does the endowment model still have merit, or are the operational complexities associated with monitoring a wider range of esoteric asset classes not justified due to reduced potential for excess risk-adjusted return?

For now, we will largely avoid the argument for mean reversion, or the concept that asset classes that perform the strongest on a trailing basis often produce more tepid results in the future. We simply note this as a possible outcome, and that the abandonment of the endowment model at this stage could constitute "selling low" in certain asset classes.

Conversely, what if, in our effort to evaluate the viability of the endowment model, we made assumptions about future asset class risk/return parameters that could be considered *detrimental* to the endowment model? Namely, that an investor constructing a portfolio on December 31, 2009 possesses some level of clairvoyance (a "crystal ball") regarding the future mean returns and standard deviations of the various asset classes she uses to construct a portfolio. In other words, the investor "knows" (to varying degrees) that asset classes such as US equities will have the favorable mean return and volatility/correlation characteristics they have realized over the past five years.

Below, we construct portfolios that would have been optimal according to a mean-variance optimizer on December 31, 2009, using information about the *probability distribution* of asset class returns from 2010-2014 (mean return, variance, covariance), for various levels of risk aversion (λ), subject to various constraints[5].

As a brief review, mean-variance optimization uses the forecast mean return, variance, and covariance of asset classes to construct portfolios that maximize the reward (expected return) at a given level of risk (variance). As we vary our risk-aversion along a continuum from complete risk-indifference to complete reward-indifference, the set of risk/reward

[5] See Appendix for full table of asset classes and respective parameters.

points corresponding to optimal portfolios forms the "efficient frontier." The symbol "λ" denotes a risk aversion level which penalizes portfolios with higher volatility[6] – a client with a higher risk aversion level will therefore prefer the optimal allocations at lower levels of risk.

There is, however, a subtle problem with this approach. It is not reasonable that anyone could have clairvoyance about the *actual* subsequent outcomes of asset class returns, but at least they are observable, ex post. It is more reasonable to allow for a degree of clairvoyance regarding the underlying *probability distribution* of returns. Unfortunately, the true probability distribution of returns is never directly observable, even after the fact. We can try to estimate the distribution, given observed asset class returns, but this is not as simple as it sounds.

We start by computing sample means, variances and correlations from the 60 monthly returns from January 2010 to December 2014. This gives a fairly noisy estimate of the joint distribution[7] (we are using relatively few observations to estimate quite a few statistics). But some asset classes are problematic. For private equity (PE) we have only quarterly returns, so we must use those (of which there are one-third as many) for the statistics for that asset class.

A more insidious problem is that of "returns smoothing." Certain asset classes, such as real estate, hedge funds, and private equity have returns that appear to be much less volatile than we might expect. This can be a consequence of lagged pricing, or deliberate manipulation of valuations/appraisals in order to produce returns that appear less volatile (benefitting Sharpe ratios).

This bias can be both detected and adjusted by various techniques[8]. We attempt to remove this kind of smoothing in the above mentioned asset classes by utilizing a simple model of smoothing[9]. Once we have incorporated this de-smoothing, and quarterly proxy returns for PE[10], we can be more confident that the sample means, variances and correlations will be a good estimate of the true distribution of returns during this 5-year period. In other words, volatility will no longer be artificially understated (and risk-adjusted returns overstated) for alternative investments.

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[6] Specifically, the amount of extra reward required to compensate the investor for one additional unit of risk.

[7] Joint distribution: probability distribution defined by the co-movement of two or more random variables. In this case the random variables are asset classes, and the joint distribution describes the "behavior" (for example, expected mean return, variance, tail risk) of a portfolio comprised of these asset classes.

[8] See, for example, Bollen, Nicolas P.B. and Veronika Krepely. "A Screen for Fraudulent Return Smoothing in the Hedge Fund Industry." Social Science Research Network, January 2006, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=686137

[9] Specifically, if S(t) is the smoothed return in period t, and R(t) is the real (unsmoothed) return in period t, then we model $S(t) = b * S(t-1) + (1-b) R(t)$. If no smoothing is present, then $b = 0$.

[10] For PE, we then use US Small Cap Equity as a proxy for PE, by fitting a relationship between the two asset classes in the history prior to December 31, 2009, and then using this relationship to derive proxy returns for PE in the 2010-2014 period. This is akin to using "matrix pricing" to price illiquid bonds. We use these proxy returns to derive the variance and correlations for PE (but not the mean).

Using the resulting estimates, we run three sets of portfolio optimizations:

1. Full clairvoyance: the investor ascribes 100% weight to future asset return distributions. In other words, the December 2009 investor only uses the mean return, variance, and covariance data from the 2010 – 2014 period to construct his/her portfolio.
2. Partial clairvoyance with historical prior: the investor ascribes 50% weight to future distributions (as discussed above) and 50% weight to 10-year trailing distributions (asset class mean return, variance, and covariance data from December 1999 – December 2009).
3. Partial clairvoyance with risk-parity prior: investor ascribes 50% weight to future distributions and 50% weight to distributions derived from the risk-parity approach, or the assumption that all asset classes have the same risk-adjusted return.

This analysis shows that even given varying degrees of clairvoyance about future asset return distributions as of December 2009, optimal portfolios still tend to incorporate aspects of the endowment model.

Initially, we assume perfect clairvoyance (i.e. the investor puts a 100% weight the "future" distribution, starting December 31, 2009), require that asset weights are non-negative (no short positions) and sum to 1.0 (no leverage). Exhibit 3 shows the (non-zero) allocations for various levels of risk aversion that result from a mean-variance optimization with no additional constraints.

Exhibit 3[11]

High Risk Tolerance

λ	ER	Vol	US LC Eqy	US SC Eqy	HF Fixed Inc	HF Eqy LS	HF Gl Macro	FI Asset Bck	FI Gvt Agncy	FI HY Muni	PE
0	16.1%	18.0%	0%	100%	0%	0%	0%	0%	0%	0%	0%
0.6	15.6%	14.1%	41%	39%	0%	0%	0%	0%	0%	0%	21%
1	15.2%	12.4%	76%	0%	0%	0%	0%	0%	0%	0%	24%
3	13.7%	9.6%	55%	0%	0%	0%	0%	0%	0%	22%	23%
5	12.0%	6.9%	35%	0%	0%	0%	0%	0%	0%	46%	19%
10	10.7%	5.3%	20%	0%	1%	0%	0%	0%	0%	63%	16%
20	8.4%	3.5%	13%	0%	13%	0%	0%	0%	29%	34%	11%
40	6.8%	2.5%	10%	0%	12%	0%	1%	0%	55%	13%	8%
100	5.6%	2.0%	8%	0%	10%	0%	3%	23%	50%	0%	5%
Inf	4.7%	1.9%	2%	0%	7%	4%	3%	55%	26%	0%	3%

High Risk Aversion

[11] See Appendix for asset class abbreviation key re: column labels.

The top row, λ=0, corresponds to being indifferent to risk, and seeks only the highest expected return (ER) portfolio. In this case, the portfolio is 100% in US small cap equity, since this asset class has the highest mean return. This is also the riskiest portfolio.

At the other extreme, infinite risk-aversion effectively eliminates consideration of ER from the optimization, and therefore the minimum risk portfolio is the result. This portfolio is predominantly allocated to fixed income asset-backed, fixed income gov't/agency, with a smattering of hedge funds, PE and US large cap equity.

A more reasonable portfolio for a typical investor might seek a risk level of around 5%. This is best approximated by the row corresponding to λ=10. This portfolio allocates to high yield munis (63%), US large cap (20%) and PE (16%), with a 1% allocation to fixed income hedge funds.

In reality, even knowing the future distribution of returns, we might still not want to risk allowing the weight in any given sub-asset class to get too big – after all, a mean return is a central tendency rather than a certainty. Exhibit 4 shows the optimal portfolios that result from imposing an upper bound of 20% on any given sub-asset class. Note that for readability, remaining exhibits show a subset of portfolios and in certain cases exclude columns with small allocations. See Appendix for complete exhibits.

Exhibit 4

λ	ER	Vol	US LC Eqy	US SC Eqy	HF Fixed Inc	HF Gl Macro	FI IG Corp	FI HY Corp	FI Gvt Agncy	FI IG Muni	FI HY Muni	PE
3	12.7%	9.2%	20%	20%	0%	0%	0%	20%	0%	0%	20%	20%
5	11.2%	6.7%	20%	9%	11%	0%	20%	0%	0%	0%	20%	20%
10	9.8%	4.9%	20%	0%	19%	4%	20%	0%	0%	3%	20%	14%
20	8.1%	3.4%	11%	0%	17%	10%	12%	0%	20%	0%	20%	10%

To obtain a similar risk level of around 5%, we can again look at the row corresponding to λ=10. This portfolio allocates 20% (the upper bound) to each of: US large cap, investment grade corporate bonds, and high yield munis; 19% to fixed income hedge funds; 14% to PE; 4% to global macro hedge funds; and 3% to investment grade munis. The important point is this: this portfolio looks more like an endowment model portfolio than a traditional 60/40 portfolio[12].

To summarize, even when assuming perfect investor clairvoyance near the beginning of the recent market cycle, mean-variance portfolios calculated for moderate levels of risk aversion tend to share similarities with endowment model portfolios (with significant allocations to alternative investments) and do not mimic traditional 60/40 portfolios. Only at the lowest

[12] "60/40 portfolio" refers to any variation of the traditional equity/bond allocation (not necessarily 60% equities and 40% bonds), with exact weightings determined by the portfolio risk target.

levels of risk aversion (i.e. riskiest portfolios) do US equities dominate the portfolio, and traditional government fixed income only appears at the highest levels of risk aversion.

Admittedly, the unrealistic assumption of uncanny foresight is not a perfect defense of the endowment model – for example, US equities are the majority of several unconstrained portfolios and there are few allocations to real assets. Perhaps we should make the fight a bit more "fair" for the endowment model. For example, the investor might decide to blend the distribution predicted by the crystal ball with the distribution corresponding to some prior set of beliefs. One reasonable (although not the only) choice of a prior is the set of means, variances and covariances estimated from the 10-year trailing realized returns. To illustrate the effect of doing this, let's assume that the investor places equal probabilities of these two distributions being the "correct" one, and so assigns equal weighting between the two. Given that these trailing 10-year returns were more favorable for endowment model asset classes, one could think of this as incorporating the "mean reversion" factor discussed earlier. Select portfolios from the resulting efficient frontier, with no constraints on allocation size, are shown in Exhibit 5.

Exhibit 5

λ	ER	Vol	Glbl Pub RE	HF Gl Macro	FI Asset Bck	FI Gvt Agncy	PE
1	10.7%	11.2%	18%	35%	0%	0%	48%
3	9.5%	6.4%	5%	75%	0%	0%	20%
5	9.2%	5.8%	2%	83%	0%	0%	15%
10	8.0%	4.2%	0%	59%	13%	16%	12%

This result includes no allocation to US Equities at any level of risk aversion, favors PE for the least risk-averse investors, global macro hedge funds for moderate risk investors, and includes government and asset-backed fixed income for more risk-averse investors. The unconstrained optimization does not produce a result that typifies the diversification of the endowment model, but does allocate to asset classes that are uniquely present in the endowment model and not available to the 60/40 portfolio.

Exhibit 6 shows select portfolios from the resulting efficient frontier if asset class weights are constrained to 20%.

Exhibit 6

λ	ER	Vol	Glbl Pub RE	US SC Eqy	Prec Mtl	HF Gl Macro	FI IG Corp	FI HY Corp	FI Asset Bck	FI Gvt Agncy	FI HY Muni	PE
1	10.3%	11.9%	20%	20%	12%	20%	0%	8%	0%	0%	0%	20%
3	8.5%	6.6%	7%	3%	4%	20%	20%	6%	0%	11%	8%	20%
5	7.8%	4.8%	2%	1%	2%	20%	20%	1%	9%	20%	4%	20%
10	7.1%	3.6%	0%	0%	0%	20%	15%	0%	20%	20%	2%	15%

At a risk aversion level of 5 (corresponding to portfolio volatility of roughly 5%), we see a portfolio comprised of the following allocation:

- 20% global macro hedge funds
- 20% investment grade corporate bonds
- 20% government fixed income
- 20% PE
- 9% asset-backed fixed income, 4% high yield munis, 2% precious metals, 2% global public real estate, 1% US small cap equity, and 1% high yield corporate bonds

Note that the precious metals asset class finds its way into this optimization at moderate to higher levels of risk aversion. Again, these allocations are far removed from a 60/40 portfolio.

As a final alternative, suppose the investor's prior set of beliefs is that all asset classes have the same Sharpe ratio, specifically 0.5. This is the key assumption that underlies the "risk parity" approach. If she knows, from her crystal ball, what the future volatilities are going to be, then she can derive future expected returns of 0.5x these volatilities[13]. The investor then blends the distribution from this risk parity approach with the distribution under full clairvoyance. The results for upper bounds on asset of holdings of 100% and 20% are shown in Exhibit 7 and Exhibit 8, respectively.

Exhibit 7

λ	ER	Vol	US LC Eqy	US SC Eqy	Prec Mtl	Agri	FI Gvt Agncy	FI HY Muni	PE
5	18.8%	11.4%	12%	15%	21%	10%	2%	16%	23%
7	15.3%	8.4%	10%	11%	14%	7%	25%	14%	19%
10	12.7%	6.3%	8%	8%	10%	5%	42%	12%	16%
15	10.7%	4.7%	6%	5%	6%	3%	55%	11%	13%

[13] Sharpe ratio = expected excess return / expected volatility

Exhibit 8

λ	ER	Vol	US LC Eqy	US SC Eqy	Prec Mtl	Agri	FI Asset Bck	FI Gvt Agncy	FI IG Muni	FI HY Muni	PE
5	18.7%	11.3%	13%	16%	20%	11%	0%	1%	0%	19%	20%
7	15.5%	8.6%	10%	11%	15%	7%	0%	20%	1%	18%	18%
10	13.1%	6.6%	10%	7%	11%	5%	0%	20%	20%	13%	15%
15	10.7%	5.0%	8%	4%	7%	4%	16%	20%	20%	10%	12%

Under the assumption of equivalent risk-adjusted returns, a given asset's diversification benefit (i.e. a low correlation to other assets in the optimization) increases in importance. The above results reflect the diversifying nature of real assets, namely precious metals and agriculture.

We have seen that, even under the assumption of perfect investor foresight regarding asset return distribution parameters (mean return, variance and covariance) and unconstrained asset weights, a mean-variance optimizer tends to allocate to asset classes such as hedge funds and private equity as opposed to a simplified allocation (such as the 60/40) that does not include endowment model asset classes. As we constrain maximum asset allocations as an appropriate additional risk control, we find that diversification into alternative investments increases. This is true even when using mean return, variance, and covariance data from 2010 – 2014, which is widely thought to have been a relatively poor period for the endowment model.

If we mix our crystal ball 50/50 (in a probabilistic sense) either with the distribution over the prior ten years, or a set of expected returns derived from an assumption of risk parity (and based on clairvoyance regarding future volatilities), we find that the optimal portfolios once again take on endowment model characteristics and also begin to include allocations to real assets.

Diversification into alternative investments tends to increase as the risk aversion assumption increases from its lowest levels, underscoring the diversifying and volatility-dampening characteristics of the endowment model approach. This is an important point: personal risk aversion levels are easily forgotten during protracted bull markets, which result in urges to concentrate portfolios excessively in top-performing but risky assets.

We have shown mean-variance efficient portfolios constructed based on a set of reasonable but limited assumptions. For example, the resulting portfolios generally do not have exposure to foreign equities. However, using different estimation techniques could have produced different allocations, such as (but certainly not limited to) modifying return expectations for the relatively low valuations in foreign equities. The analysis is also pre-tax and does not adjust returns for varying client tax situations. While these issues should be considered closely in constructing specific Investment Policy Statements, further specifying our optimization methodology is beyond the scope of this paper, the objective of which is to evaluate endowment-style portfolios relative to a simple, traditional allocation like the 60/40 (rather than recommending a particular optimal portfolio for investors).

An investor should appropriately start with a set of prior assumptions regarding return distribution parameters, and then modify these in light of recent experience, and/or other information that comes into her possession. This is highly unlikely to lead to a 60/40 portfolio, even under the assumption of relatively strong performance for US equities, which account for most of the variation in 60/40 portfolio return. The optimality of the 60/40 approach implies that all other asset classes must somehow have been perceived to have poor expected returns, so that a mean/variance optimizer would not include them *at all* in her portfolio.

But we see that, even with varying degrees of clairvoyance regarding the future probability distribution of asset returns (as opposed to clairvoyance regarding actual outcomes, which is surely too much to ask), an endowment model-style portfolio is still to be preferred to a traditional 60/40 portfolio.

Even taking its recent challenges into account, the endowment model still appears to be an attractive approach to managing assets in a long-term, risk-conscious manner.

Appendix: Asset Class Universe

We consider the following set of sub-asset classes:

1. US Large Cap Equity (US LC Eqy)
2. US Small Cap Equity (US SC Eqy)
3. Developed non-US Large Cap Equity (Dev LC Eqy)
4. Developed non-US Small Cap Equity (Dev SC Eqy)
5. Emerging Markets Large Cap Equity (EM LC Eqy)
6. Emerging Markets Small Cap Equity (EM SM Eqy)
7. Global Public Real Estate (Glbl Pub RE)
8. Energy (Energy)
9. Precious Metals (Prec Mtl)
10. Industrial Metals (Ind Mtl)
11. Agricultural Commodities (Agri)
12. Multi-strategy Commidities (Mlti Strt Comm)
13. Hedge Funds in Fixed Income (HF Fixed Inc)
14. Hedge Funds in Equity Long/Short (HF Eqy LS)
15. Hedge Funds in Event Driven (HF Event Driven)
16. Hedge Funds in Global Macro (HF Global Macro)
17. Fixed Income in Investment Grade Corporates (FI IG Corp)
18. Fixed Income in High Yield Corporates (FI HY Corp)
19. Fixed Income in Asset Back Securities (FI Asset Bck)
20. Fixed Income in Government/Agency Securities (FI Gvt Agncy)
21. Fixed Income in Foreign Debt (FI Frgn Debt)
22. Fixed Income in Investment Grade Municipal Bonds (FI IG Muni)
23. Fixed Income in High Yield Municipal Bonds (FI HY Muni)
24. Private Equity (PE)

A table showing sub-asset class parameters, ranked by 2010-2014 returns, is shown on the following page.

	2010-2014		2005-2009	
	Average	**Std. Dev.**	**Average**	**Std. Dev.**
US SC Eqy	16.12%	17.98%	2.83%	21.35%
US LC Eqy	15.28%	13.00%	1.73%	16.05%
PE	15.06%	6.20%	8.62%	13.43%
Glbl Pub RE	12.72%	16.28%	5.39%	25.54%
Dev SC Eqy	10.24%	17.06%	6.75%	22.18%
FI HY Corp	8.87%	6.30%	7.19%	13.46%
FI HY Muni	8.24%	5.59%	3.06%	9.72%
Dev LC Eqy	7.04%	16.69%	5.93%	19.67%
HF Fixed Inc	7.01%	2.05%	1.02%	9.19%
HF Eqy LS	6.46%	7.31%	6.64%	8.68%
FI IG Corp	6.39%	4.11%	4.76%	7.36%
HF Gl Macro	6.21%	3.65%	9.00%	6.64%
HF Event Driven	5.40%	7.10%	7.89%	6.98%
FI IG Muni	5.11%	3.79%	4.35%	4.87%
EM LC Eqy	3.79%	18.54%	18.85%	27.99%
FI Asset Bck	3.74%	2.20%	5.69%	3.11%
FI Gvt Agncy	3.69%	3.09%	4.86%	4.32%
Prec Mtl	2.88%	22.12%	20.13%	22.82%
EM SC Eqy	2.49%	18.60%	18.84%	30.66%
Agri	1.83%	23.32%	7.14%	24.07%
FI Frgn Debt	1.23%	6.89%	4.67%	8.72%
Mlti Strt Comm	-4.45%	15.65%	4.06%	20.41%
Ind Mtl	-4.49%	21.21%	16.97%	27.23%
Energy	-14.37%	20.52%	-7.44%	35.28%

Appendix: Full Optimization Results

Exhibit 4

λ	ER	Vol	Dev SC Eqy	Glbl Pub RE	US LC Eqy	US SC Eqy	HF Fixed Inc	HF Eqy LS	HF Gl Macro	FI IG Corp	FI HY Corp	FI Asset Bck	FI Gvt Agncy	FI IG Muni	FI HY Muni	PE
0	13.9%	14.4%	20%	20%	20%	20%	0%	0%	0%	0%	0%	0%	0%	0%	0%	20%
1	13.5%	11.4%	0%	20%	20%	20%	0%	0%	0%	0%	0%	0%	0%	0%	20%	20%
3	12.7%	9.2%	0%	0%	20%	20%	0%	0%	0%	0%	20%	0%	0%	0%	20%	20%
5	11.2%	6.7%	0%	0%	20%	9%	11%	0%	0%	20%	0%	0%	0%	0%	20%	20%
10	9.8%	4.9%	0%	0%	20%	0%	19%	0%	4%	20%	0%	0%	0%	3%	20%	14%
20	8.1%	3.4%	0%	0%	11%	0%	17%	0%	10%	12%	0%	0%	20%	0%	20%	10%
40	6.9%	2.6%	0%	0%	7%	0%	16%	0%	12%	0%	0%	20%	20%	0%	18%	7%
100	5.9%	2.3%	0%	0%	4%	0%	17%	0%	14%	0%	0%	20%	20%	19%	0%	5%
Inf	5.6%	2.2%	0%	0%	0%	1%	18%	1%	16%	0%	0%	20%	20%	20%	0%	4%

Exhibit 5

λ	ER	Vol	Glbl Pub RE	HF Event Driven	HF Gl Macro	FI Asset Bck	FI Gvt Agncy	FI HY Muni	PE
0	11.8%	17.3%	0%	0%	0%	0%	0%	0%	100%
0.3	11.8%	16.3%	28%	0%	0%	0%	0%	0%	72%
0.6	11.8%	16.3%	29%	0%	0%	0%	0%	0%	71%
1	10.7%	11.2%	18%	0%	35%	0%	0%	0%	48%
3	9.5%	6.4%	5%	0%	75%	0%	0%	0%	20%
5	9.2%	5.8%	2%	0%	83%	0%	0%	0%	15%
10	8.0%	4.2%	0%	0%	59%	13%	16%	0%	12%
20	6.8%	3.0%	0%	0%	32%	54%	4%	0%	9%
40	6.2%	2.6%	0%	0%	19%	73%	0%	2%	7%
100	5.8%	2.5%	0%	3%	9%	81%	0%	2%	5%
Inf	5.5%	2.4%	0%	5%	1%	87%	0%	3%	4%

Exhibit 6

λ	ER	Vol	Dev SC Eqy	Glbl Pub RE	US LC Eqy	US SC Eqy	Prec Mtl	HF Event Driven	HF Gl Macro	FI IG Corp	FI HY Corp	FI Asset Bck	FI Gvt Agncy	FI IG Muni	FI HY Muni	PE
0	10.6%	13.9%	20%	20%	0%	20%	0%	0%	20%	0%	0%	0%	0%	0%	0%	20%
0.3	10.6%	13.9%	20%	20%	0%	20%	0%	0%	20%	0%	0%	0%	0%	0%	0%	20%
0.6	10.5%	12.7%	7%	20%	0%	20%	13%	0%	20%	0%	0%	0%	0%	0%	0%	20%
1	10.3%	11.9%	0%	20%	0%	20%	12%	0%	20%	0%	8%	0%	0%	0%	0%	20%
1.6	9.7%	9.9%	0%	20%	0%	10%	8%	0%	20%	19%	3%	0%	0%	0%	1%	20%
2.3	9.1%	8.0%	0%	11%	0%	5%	6%	0%	20%	20%	8%	0%	0%	0%	10%	20%
3	8.5%	6.6%	0%	7%	0%	3%	4%	0%	20%	20%	6%	0%	11%	0%	8%	20%
5	7.8%	4.8%	0%	2%	0%	1%	2%	0%	20%	20%	1%	9%	20%	0%	4%	20%
10	7.1%	3.6%	0%	0%	0%	0%	0%	0%	20%	15%	0%	20%	20%	8%	2%	15%
40	6.5%	3.1%	0%	0%	0%	0%	0%	4%	20%	6%	0%	20%	20%	20%	3%	7%
Inf	6.3%	3.1%	0%	0%	1%	0%	0%	7%	20%	4%	0%	20%	20%	20%	5%	4%

Exhibit 7

λ	ER	Vol	Glbl Pub RE	US LC Eqy	US SC Eqy	Energy	Prec Mtl	Agri	HF Event Driven	HF Gl Macro	FI Asset Bck	FI Gvt Agncy	FI HY Muni	PE
0	23.6%	19.9%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
3	21.8%	14.2%	8%	0%	25%	0%	24%	12%	0%	0%	0%	0%	0%	31%
5	18.8%	11.4%	0%	12%	15%	0%	21%	10%	0%	0%	0%	2%	16%	23%
7	15.3%	8.4%	0%	10%	11%	0%	14%	7%	0%	0%	0%	25%	14%	19%
10	12.7%	6.3%	0%	8%	8%	0%	10%	5%	0%	0%	0%	42%	12%	16%
15	10.7%	4.7%	0%	6%	5%	0%	6%	3%	0%	0%	0%	55%	11%	13%
Inf	5.0%	2.4%	0%	0%	0%	0%	0%	0%	5%	2%	86%	0%	3%	4%

Exhibit 8

λ	ER	Vol	Glbl Pub RE	US LC Eqy	US SC Eqy	Prec Mtl	Agri	HF Event	HF Gl Macro	FI IG Corp	FI Asset Bck	FI Gvt Agncy	FI IG Muni	FI HY Muni	PE
0	22.0%	14.8%	20%	0%	20%	20%	20%	0%	0%	0%	0%	0%	0%	0%	20%
3	21.5%	14.1%	12%	13%	20%	20%	16%	0%	0%	0%	0%	0%	0%	0%	20%
5	18.7%	11.3%	0%	13%	16%	20%	11%	0%	0%	0%	0%	1%	0%	19%	20%
7	15.5%	8.6%	0%	10%	11%	15%	7%	0%	0%	0%	0%	20%	1%	18%	18%
10	13.1%	6.6%	0%	10%	7%	11%	5%	0%	0%	0%	0%	20%	20%	13%	15%
15	10.7%	5.0%	0%	8%	4%	7%	4%	0%	0%	0%	16%	20%	20%	10%	12%
Inf	6.5%	3.0%	0%	0%	0%	0%	0%	7%	20%	4%	20%	20%	20%	5%	4%

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